# Motivating Inquiry in Statistics and Probability in the Primary Classroom

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**Summary** We describe how the use of a games environment combined with technology supports upper primary children in engaging with a concept traditionally considered too advanced for the primary classes: *The Law of Large Numbers*.

**Keywords:** Teaching; Law of Large Numbers; Primary mathematics; Initial teacher education; Statistics and probability; Simulation.

## INTRODUCTION

Understanding statistics and probability can be a challenge for children and adults. Children's everyday experiences and intuitions about probability often pose obstacles to developing correct understanding of probabilistic concepts (Fischbein 1975; Shaughnessy 1992). The goal of this study was to support primary school students in collecting data and in understanding the *Law of Large Numbers*.

Bernoulli's Law of Large Numbers states that as the number of trials of a random process increases, the percentage difference between the expected and actual values goes to zero. Research indicates that adults (Tversky and Kahneman 1974; Fischbein and Schnarch 1997) and children (Ireland and Watson 2009; Konold et al. 2011) hold a range of misconceptions relating to concepts that underpin the Law of Large Numbers. For more in-depth discussion of this research, we direct you towards Falk and Lann's (2015) work in this area, which provides a comprehensive account of the Law of Large Numbers and an overview of the related research. Our research focuses on developing understandings of primary level students. Hence, the description of the Law of Large Numbers we draw upon is an informal one, i.e. the greater the number of trials, the closer the experimental results will be to the theoretical probability. Although in the Irish curriculum probability or *chance* is introduced from the 3rd grade, and the curriculum objectives for grade 6 recommend that students *should perform the experiment a large number of times*, there is no reference to the formal introduction of this concept. In the USA, the Common Core Standards make no reference to probabilistic concepts until grade 7. Our research was motivated by an interest in whether children in upper elementary school demonstrate a readiness for this concept.

## RESEARCH CONTEXT

### Participants

This study describes research carried out in a college of education in Ireland. The college graduates approximately 50% of Irish pre-service primary teachers. Twenty final-year pre-service primary teachers participated in the study during the concluding semester of their teacher education programme. Participants had completed their mathematics education courses (three semesters) and all teaching practice requirements (at junior, middle and senior grades) and self-selected into mathematics education as a cognate area of study.

### Research design

In this study, pre-service teachers and two mathematics educators used *Japanese Lesson Study* (Fernandez and Yoshida 2004; Lewis and Tsuchida 1998) to examine the planning and implementation of probability lessons in primary classrooms. Lesson study is increasingly being used in Initial Teacher Education to explore the impact of different instructional approaches on the development of children's mathematical understanding (Leavy et al. 2013; Leavy 2010; Murata 2011).

Pre-service teachers worked in four small *lesson study groups* on the design and implementation of four different *study lessons*. The process around the design and teaching of these study lessons involved the following phases:

Phase 1: This phase involved the *research and preparation* of a study lesson. Each group of pre-service teachers was presented with one concept in probability that they researched and explored using research literature, curriculum documentation and other resources provided by the researchers. These topics were identified from a review of international curricula and recommendations from professional organizations (e.g. Guidelines for Assessment and Instruction in Statistics Education). The researchers acted as mentors and supported pre-service teachers in using the research outcomes and recommendations to inform the design of a detailed lesson plan for use in a primary classroom. The lesson format adhered to guidelines put forward by Ertle et al. (2001) and incorporated specific reference to steps of the lesson (learning activities and key questions), student activities, expected student responses, teacher response to student activity/response, and goals and methods of evaluation.
Phase 2: The *implementation* stage involved one pre-service teacher from each group teaching the lesson in a 5th grade classroom while the remainder of the group and the researchers observed and evaluated classroom activity and student learning. Subsequently, following discussion, the original lesson design was modified in line with their observations in an effort to improve the learning outcomes for children. The *second implementation* stage involved teaching the revised lesson with a second different class of 5th grade students and *reflecting* upon observations. The second implementation was videotaped. It is this second lesson that is described in this article.
Phase 3: This phase represents the conclusion of the lesson study cycle and involved each lesson study group making a presentation of the outcomes of their work to their peers and lecturers at the end of the semester.

The process of lesson study facilitated the design of tools and sequences of instruction to support the development of statistical and probabilistic reasoning with primary children. The study lesson we describe has been designed, taught and revised through the lesson study process where we were working with 5th graders (aged 10–11 years old) in two different primary schools. Whereas we focus here on the final lesson in the week sequence, i.e. the Law of Large Numbers, the previous lessons addressed probability concepts in the following sequence: describing likelihoods, comparing and explaining likelihoods and ordering likelihoods of events.

## THE LESSON SEQUENCE

We wanted to create an environment that fostered children's curiosity regarding probability and engage them in discovery-based, hands on inquiry in mathematics. In the series of activities we describe here, we used the context of simple carnival games to explore the relationship between experimental and theoretical probability. Two technologies were used to motivate the inquiry: video and the National Council of teachers of Mathematics (NCTM) illuminations online applet.

### Caine's Arcade

As we were eager to situate our explorations of probability within a relevant and real world context for children, we decided to begin the lesson by showing the children a video clip of Caine's Arcade (http://cainesarcade.com/ (figure 1)). This video narrates the story of a 9-year-old boy who spent his summer building an elaborate cardboard arcade inside his dad's auto parts store. In the video excerpt Caine shows his arcade games and describes how some are easier to win than others. He also reports that he uses his own toy cars as prizes. However, as he only has a certain number of cars, and if every customer wins every game every time, he would run out of prizes very quickly.

After watching the video, 5th grade children discussed how Caine analysed the games so that they would not be too easy. For example, in the football game, children acknowledged that 'Caine added some goalies' in an effort to


**Fig. 1.** Caine's Arcade

'make it harder to win'. Then, we presented the class with this challenge:

> *The challenge*: Caine is always adding new games to his arcade. We know that he needs games that are possible to win. However, the chances of winning cannot be so high that he runs out of prizes. By the end of today, you are going to recommend a game that would be suitable for his arcade. To do this well, we need to experiment and explore lots of different games to find out more about chances of winning so we can help Caine.


**Fig. 2.** Recording sheet

### Launching the lesson: exploring fairness

The lesson started with an introductory probability activity. This activity was selected to provide practice in recording probabilities, to revise the language of uncertainty and engage in a discussion of fairness. The teacher showed a bag with six counters. Three of the counters were yellow, and three were red. The teacher mixed up the counters in the bag and said he was going to select one counter. Children were informed that if they selected a yellow counter, they would *win*, and if they selected a red counter, they would *lose*. He then posed a series of questions and allowed children to work in small groups to discuss and record their answers (the recording sheet was displayed on the board, see figure 2). After each question, the teacher provided opportunities for groups to report their findings. In all cases, students were expected to justify their responses. The teacher asked the following questions:

- What are the possible outcomes? What colour might the counter be?
- What are the chances of choosing a yellow counter? Why?
- Do you think this is a fair game? What do you think I mean by a 'fair game'?

Children were then invited to use all knowledge and conclusions to date to predict the experimental probability:

- Let us agree that this is a fair game and the chances of choosing a yellow counter are 50 : 50 or ½ of the time. If I play the game in exactly the same way 6 times, how many times do you think I will choose a yellow counter?

Following teacher modelling of the activity, children were organized into small groups and played the game 6 times. Each time they were instructed to shake the bag, reach in without looking, choose a counter and record its colour on their record sheet and then put the counter back into the bag and shake the bag once again (figure 3). When they had played 6 times and recorded their results, they added up their totals for red and yellow.

The following discussion occurred with one group:

Teacher: If I were to do this six times, how many yellows would you expect to get?
Rebecca: One yellow the first time but three yellows altogether.
Mark: Three yellows.
Teacher: Why do you say that Mark?
Mark: Because there are 3 yellows and 6 counters altogether.
Cian: Well you might get 3 yellows. But you do not have to. You could get 4 reds and 2 yellows.

*Children engaged in the activity. Six red counters (zero yellow counters) were removed.*

Teacher: So our prediction was 3 yellows, and we got 6 reds. Do you have any idea why that may have happened?
Rebecca: We were just unlucky.
Teacher: If we were to play it again, would we get the same result?
Rebecca: No. We might not be unlucky that time.
Teacher: Is this a fair game?
Alan: Well, there are 3 reds and 3 yellows. So, you could get a red or a yellow, but we are not guaranteed to lose or to win.
Teacher: Do you think this game favours the arcade owner (Caine) or the player?
Cian: The arcade owner

As this conversation illustrates, the results from the activity (i.e. losing the *fair* game) caused some children to conclude that the game was unfair (i.e. the game favoured the arcade owner).

Following the small group activity, a brief whole-class discussion occurred during which the following questions were posed to the children.

**Fig. 3.** Introductory tile game activity

- Looking at our results, did any group draw yellow 3 out of 6 times as we expected?
- Did any group get a result they were not expecting?
- Do we agree that the game is fair? Why, then, did groups get different results?
- How can we carry out our investigation differently, or arrange our results differently, so that the outcome is closer to what we expect it to be (50 : 50)?

Subsequently, the children were asked to share their outcomes with the class. The results for each group were entered into a spread sheet (displayed on the interactive whiteboard), and the total for the class was calculated. Therefore, although some of the groups had experienced large discrepancies between their theoretical (3 yellow and 3 red) and experimental probabilities (0 yellow and 6 red (figure 4)), they were exposed to the fact that when only a small number of trials are used, it is difficult to predict the outcome. Children were asked to consider that perhaps, when the game is played more times, 'what actually happens moves closer to what we expect to happen'. In other words, from a sample of 36 draws from the bag, there were 19 yellow and 17 red tiles removed, i.e. almost half (figure 5). Therefore, pupils were asked to consider that perhaps increasing the number of times we play the game gives us a better indication of how the game works, i.e. the Law of Large Numbers.

### Exploring probability using stations

Children were informed that they would play some carnival games at a series of stations – with the goal of identifying a game that would be suitable for Caine's arcade. Each station had a sign stating the objective of the game or what was required to *win* in the game. There was a student teacher at each station who coordinated the activity. By examining the materials and the objective, children were encouraged to determine the probability of winning each game (and record it on their activity sheet), before actually playing the game. After playing each game the assigned number of times, the group was encouraged to use their findings to decide if the particular game was suitable for Caine's arcade, i.e. how many toys Caine might have to give away if this game featured in his arcade. Each group of children rotated through all the stations, and they recorded the predicted and actual outcome of each activity.


**Fig. 5.** Whole-class pooled data from the introductory tile game activity

| | | | | | |
|---|---|---|---|---|---|
| Grúpa 1 | Win | Lose | Grúpa 4 | Win | Lose |
| Total | 4 | 2 | Total | 6 | 0 |
| Grúpa 2 | Win | Lose | Grúpa 5 | Win | Lose |
| Total | 3 | 3 | Total | 1 | 5 |
| Grúpa 3 | Win | Lose | Grúpa 6 | Win | Lose |
| Total | 2 | 4 | Total | 3 | 3 |

**Fig. 4.** Group outcomes from the introductory tile game activity

Station 1: *The 12-sided die*

At this station, the group had to roll a 12-sided die; each side was numbered 1–12. They must roll a 1 to win. They first calculated the probability of winning (1 in 12) and then rolled the die 12 times and recorded how many times they rolled a 1.

Station 2: *Find the King*

The group were presented with 3 cards. One of the cards was a King – you must find the king to win. The cards were shuffled and then placed face down on the table. Children calculated the probability of winning (1 in 3) and then played this game 3 times and recorded how many times that they found the King.

Station 3: *Bag of Cards*

This station had a bag with 9 cards contained within it. There were 2 Aces and 7 other cards. The children had to pick a card from the 9 cards hidden in a bag. They must pick an Ace to win. They calculated the probability of winning (2 in 9) and then played this game 9 times, replacing the selected card each time. The children recorded how many times they found an Ace.

Station 4: *Bingo*

In this activity, a bag contained 12 counters: 10 red and 2 green. The children had to draw a red counter to win. The children calculated and recorded the chance of winning (10 in 12). They took a counter from the bag, made a record of the colour and then replaced it. They made 12 draws. The children recorded how many times they drew a red counter.

Station 5: *Spinner*

This activity used an electronic spinner located on the NCTM Illuminations website [http://illuminations.nctm.org/ActivityDetail.aspx?ID=79]. In the spinner game, the region (a circle) was divided into 12 wedges. The spinner had to land on a wedge in the top half to win. Students recorded the chance of winning (6 in 12) and carried out 12 spins. They recorded how many times the spinner landed in the top half of the circle.

Station 6: *Lottery*

A bag was filled with 9 numbered ping-pong balls, each numbered with one of the digits 1–9. Students must draw a 9 to win. They recorded the chances of winning (1 in 9) before making 9 draws. They then recorded the number of times they drew a ping-pong ball with the number 9.

## Finding the best game for Caine's Arcade: what happens when we increase the number of trials?

When all groups had completed a rotation of all stations, we engaged children in a discussion of the probability of winning the games.

| | |
|---|---|
| Teacher: | Which game should have been the easiest to win? |
| Evan | 'Find the King' |
| AJ: | 'Bingo' |

*Children nod their heads in agreement*

| | |
|---|---|
| Teacher: | What do you think Sophie? |
| Sophie: | The 'Find the King' game and the 'Bag of Cards' |
| Teacher: | Did you win any game more times than you expected? |
| Evan: | We won 'Find the King' 3 times. |
| Teacher: | How many times did you expect to win it? |
| Evan: | Only 1 time. But we got the king 3 times. |
| Adam: | For us, it was the 'Bag of Cards' game. We thought we would get 2 in 9, but we got 5 aces. |
| Teacher: | How could we play the game and get closer to the expected result? |
| Anna: | We could remove some of the cards or balls. |
| Amanda: | We could play it lots and lots. |

As we can see from the discussion above, some children suggested that the more times you carry out the activity, the closer your actual results are to the expected/predicted results. Using the students' idea as a springboard, the teacher suggested that if we play each game more times, as they did with the initial counter activity, we might get a better idea of whether or not it is a good game for Caine's Arcade. He told the children that they were going to investigate this phenomenon some more. Each group was invited to revisit their first station and play that game 36 times. At that station they were reminded of their results the first time and then asked to predict their results if they played the game 36 times. Once finished, they compared their results with the first time they played the same game. In some cases, it was necessary to address 'equivalent fractions' through the use of a fraction wall (a pedagogical tool commonly found in primary classrooms) in order for the comparison to take place. We used the following questions to guide both the small group and ensuing class discussion:

- What differences did you notice between the first time you played the game and when you played it 36 times?
- Which results were closer to the chances that we expected: the first time or the second time? Why do you think that is? Discuss this in your groups and see if you can come up with a reason for your answer.

The following is an excerpt from the whole-class discussion:

| | |
|---|---|
| Teacher: | Which results were closer to the chances that we expected: the first time or the second time? |
| Children: | The second time |
| Teacher: | Why do you think that is? |
| Sarah: | We tried it more. |
| Anna: | We had more luck. |
| Niamh: | We had more tries. |
| Ailbhe: | Yeah, we had more chances of doing it. |

The majority of the groups found that when they played the game 36 times, the actual amount of times they won the game was closer to what they expected as compared with when they played the game the first time. The teacher told them that they were going to explore the hypothesis that *the outcome is more reliable/ predictable when you carry out the activity a large number of times* (in other words, the experimental probability moves closer to the theoretical probability).

We used the spinner game (explored at station 5) to explore the hypothesis. We selected the use of technology as it allowed us to explore a large number of trials in a very short amount of time and provided a visual representation of the events. The two sets of data (results from 12 spins and results from 36 spins) from the spinner group were presented to the class on the interactive white board (figure 6) and discussed the results. Through the use of a pie chart to represent the results of the two data sets, children saw that in the smaller number of trials, they won the game 4 out of 12 times; whereas in the larger number of trials, they won the game 15 out of 36 times. Students noticed that at the larger trial, the actual results were closer to, but not exactly the same as, the predicted results.

The teacher then told the class that they were going to try a really large test and use the computer to spin 1000 times. He asked them how many times they would expect to win. Students predicted that it would be close to 500 times, and there was some student-initiated discussion relating to the role that luck would play. We used the Illuminations online spinner and completed 1000 spins and placed the new results on the board alongside the original investigations (figure 7). As we can see, in the third computer-simulated investigation, students won 506 times and lost 494 times. A discussion was held regarding the outcomes, motivated by the following discussion questions:

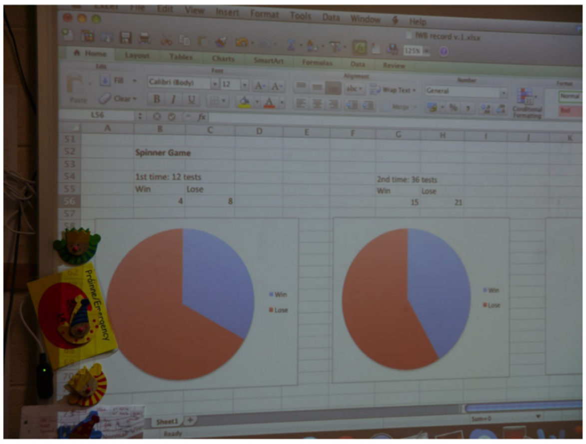


**Fig. 6.** Exploring the spinner results for two investigations

- When we spun 1000 times, how many times did we win?
- In your groups, compare our large test with our small test and see which one was closer to the expected value of 50 : 50.
- The outcome of which test was closer to our expected chances of winning?

| | |
|---|---|
| Teacher: | When we spun 1000 times, how many times did we win? |
| Emily: | 506 |
| Teacher: | Was that close to our prediction? |
| Emily | We predicted 500. It is not perfect, but it is close. |
| Teacher: | So what happens as we increase the number of times we play? |
| Katie: | We get closer. |
| AJ: | Yeah, we get closer. |

To conclude the activity, the opportunity was taken to revisit the video of Caine's Arcade. Students had little difficulty determining which of

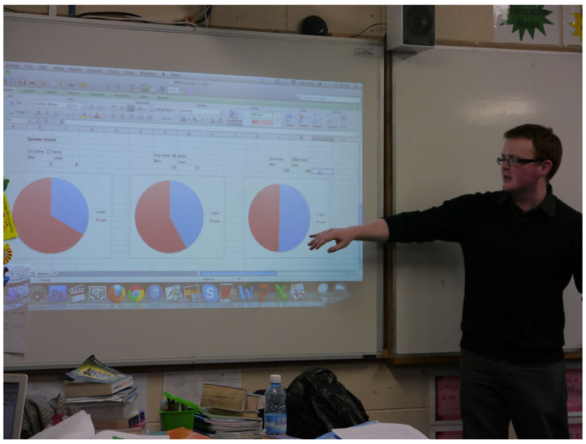

**Fig. 7.** Comparing the spinner results as number of trials increases to 1000

our activities would be best for Caine to use in the arcade. They also readily shifted perspective and discussed the best game for an arcade player to play drawing upon the factors of *easy to win* and *enjoyment value* on which to base their decisions. Overall, we found that students were able to confidently and competently utilize the language of probability in these final discussions, could readily identify the difference between theoretical (expected) and experimental (actual) results and applied their new knowledge of experimental probability to approximate the theoretical probability in the final task.

## Reflections

We believe that the context of Caine's Arcade supported the meaningful exploration of probabilistic concepts situated within a real world situation. The opportunity to play various carnival-style games, each designed to represent different outcomes, helped primary children become aware of the shortcomings of a small number of trials. In general, children demonstrated a readiness to consider that completing the game a large number of times would provide Caine with the best feedback of the chances of winning/losing the game – as he needed to think of long-term outcomes. The majority of children could see that it was sometimes difficult to predict the outcome when using a small number of trials. However, the concepts underpinning the Law of Large Numbers are cognitively challenging for young learners. For some children, their focus on the role played by *luck* provided an obstacle in terms of making judgments about the possible outcomes of games. However, the Illuminations spinner greatly supported children in exploring the Law of Large Numbers and lead them to discover, through simulation, that the number of trials impacts the relationship between theoretical and experimental probability, i.e. what we expect and what actually happens move closer together as the number of trials increases.

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