Original Article

Crime scenes and mystery players! Using driving questions to support the development of statistical literacy

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Abstract We argue that the development of statistical literacy is greatly supported by engaging students in carrying out statistical investigations. We describe the use of driving questions and interesting contexts to motivate two statistical investigations. The PPDAC cycle is use as an organizing framework to support the process statistical investigation.

Keywords: school statistics; distribution; statistical literacy; Teaching.

Our goal for the study of statistics in school mathematics is to develop statistically literate young adults who make reasonable assumptions when presented with quantitative data. Too often, we find that while students develop adequate procedural knowledge of how to calculate statistics and construct graphs, they are less able to reason about when and where to use these measures. Hence, students may perceive statistics as a set of discrete procedures and possess limited functional understandings of the interconnections between elements of statistical investigation.

Developing conceptual understanding alongside functional literacy

Advances in statistics education have led to the emergence of tools and pedagogies supporting the development of conceptual understanding. Take the mean, for example, there are multiple approaches and technologies that promote conceptual understanding by developing fair share, levelling out or balance models of the mean. However, a fundamental problem remains that even where conceptual understanding exists, when presented with data, school students often do not recognize situations when the mean is a useful measure to use (e.g. when comparing distributions of unequal sample size). Such instances indicate poor functional literacy of statistics.

Developing statistical literacy

The unique skills and ways of thinking associated with statistics can be developed by bringing learners through the process of statistical investigation. In this article, we present a series of classroom-tested investigations carried out with 12 year olds that situate the teaching of statistics within the context of statistical investigation. We illustrate how engaging students in *statistical investigations* reveals the unique capabilities of graphs and statistical measures to reveal and summarize aspects of their data and thus support the development of functional literacy of statistics.

USING THE PPDAC CYCLE TO STRUCTURE INVESTIGATIONS

The investigations we describe were designed, taught, modified and retaught to 6th-grade students using the process of Lesson Study (Hourigan and Leavy, 2016; Leavy, 2010, 2015; Lewis and Tsuchida 1998; Stigler and Hiebert 1999). We structure our statistics lessons using the problem, plan, data, analysis and conclusion (PPDAC) investigative cycle (Wild & Pfannkuch, 1999). The first step of the PPDAC cycle involves the identification of a *problem,* presented as a driving question, which generates curiosity and motivates students to want to collect data. The *planning* phase involves identifying the data collection procedures best suited to the investigation and making predictions about the outcome. The collection and organization of data constitute the *data* phase and the ensuing *analysis* phase

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engages learners in the analysis of data through observation of patterns and trends in the data, the construction of graphs and calculation of statistical measures of centre and variability. The cycle is brought to a close in the *conclusion phase* where students make conclusions about what has been learned from the investigation.

A focus on distribution

In the statistical investigations we describe, critical opportunities to develop statistical literacy are embedded within the analysis of distributions of data. The data used to construct the distributions are collected in response to the research question (figure 1). Distributions are then communicated on graphs – which serve as important reasoning tools. The analysis of these distributions requires the use of the statistical measures underpinning middle school statistics: shape, centre, variability, sampling and informal inference.

INVESTIGATION 1: CRIME SCENE INVESTIGATION IN THE CLASSROOM

Students act as *data detectives* in this exploration of the characteristics of a data distribution collected as part of a fictitious crime scene investigation. The emphasis on describing and summarizing the distribution necessitates a focus on statistical measures to accurately depict the statistical distribution.

Fig. 2. Shoeprint found at location of crime scene

Box 1
Targeted Statistical Understanding: [US: CCSSMATH.6.SP.A.2; UK: Key Stage 3]
The data collected to answer the statistical question have a *distribution*. The data values that are part of this distribution will vary. This variation can be described by referring to its shape or measures of centre and spread.

- We may use the centre and spread of specific distributions to eliminate students as suspects in a fictitious crime scene (investigation 1) or to predict to which team a player belongs (investigation 2).
- The distributions we construct may be symmetrical or skewed and may have landmarks such as gaps or outliers. This shape can be used for comparison (investigation 2).

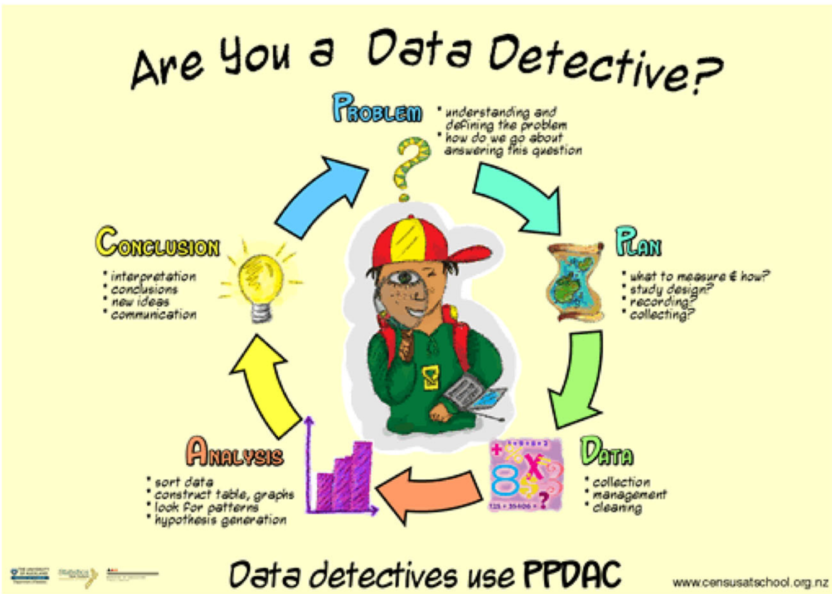

Fig. 1. The PPDAC cycle (from censusatschool.org.nz)

PPDAC: problem

The teacher explained that a burglary happened in the school, and the only evidence the police crime investigation bureau found was a shoe print (figure 2) in the flowerbed. Their goal was to investigate whether the shoe print belongs to a 6th grader and hopefully eliminate themselves as suspects.

Box 2
Targeted Statistical Understanding: [US: CCSSM. 6.SP.B.5B; UK: Key Stage 3]
The planning component of the investigation necessitated that students describe the nature of the attribute under investigation (shoe size rather than foot size), including how it was measured (the technique involved) and the unit of measurement (centimetres).

PPDAC: plan

Students discussed ways to rule themselves out as suspects. Strategies included gathering alibis for the night in question and making sworn statements. They reached agreement that they should measure each shoe print and compare it with the police evidence. They developed a common procedure and unit of measurement (centimetre) to ensure accurate measurement.

Box 3
Targeted statistical understandings: [CCSSM. 6.SP.A.3; UK: key stages 2 and 3]
Distributions of data can be described by generating summary measures that describe the centre of a distribution and by generating a measure of variation, which describes how the data values vary.

Fig. 3. Measuring the shoe print

Rachel (Group 1)	We predict that our shoe size falls between 24 and 28 cm because this includes the biggest and smallest shoe sizes in our group [variability]
Niamh (Group 2)	We think 26 cm is a good value. We chose this because it uses all our measurements – we added our shoe lengths and divided by 4 [centre]

PPDAC: data

Each student traced, measured and recorded their shoe length (figure 3). Groups then used their individual measurements to identify a shoe length, which best represents the class. These descriptions provided insights into the distributional features that were salient for them. The teacher discussed the affordances of different strategies referring to ranges and means as summarizing the distribution in just one value.

PPDAC: analysis

Analysis involved exploring the characteristics of the class distribution of shoe sizes. Each student placed their data value on a line plot (figure 4), and the teacher posed a series of questions focusing on the characteristics of the distribution. Questions increased in complexity and were structured to facilitate students in reading the data, reading between the data and reading beyond the data (Curcio 1987). Questions involving 'reading the data' are low in cognitive demand and involve taking information directly off the graph, i.e. identifying the most frequently occurring data value. Requiring students to 'read between the data'

Box 4
Targeted statistical understanding: [CCSSM.6. SP.B.4; UK: key stage 3]
This activity develops the understanding that graphs function as communication tools. Graphs communicate aspects of distributions by illustrating patterns and trends in the data.

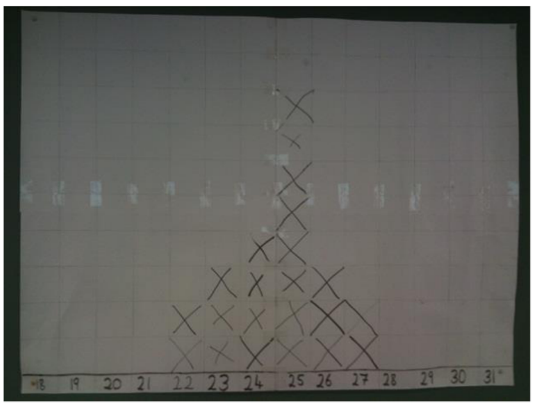

Fig. 4. Class distribution of shoe sizes

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involves some level of interpretation of the data and requires one step to solve, i.e. identifying the total number of data values or range of data values. Finally, 'reading beyond the data' involves making predictions or informal inferences about the data, i.e. making inferences about the population based on the sample. This final category of questions provided insights into students' statistical reasoning.

Students initially worked in groups to identify interesting features of the distribution of data.

This facilitates them to begin to 'read the data' and 'read between the data'.

Sarah	25 cm is the most popular shoe length [frequency]
Declan	The graph goes from 22 to 27 [range]
Tom	It looks like the Eiffel Tower [distributional shape]

Their responses were used to guide a formal analysis of the *distribution*. For example, the teacher reminded students that the statistical term for the 'most popular/common' is the *mode* (25 cm) and the term *range* is used to describe the interval from the lowest to the highest data values. The shape of their distribution was discussed with specific reference to the *cluster of data* (naturally occurring group of values) and the absence of *outliers* (unusual data values separated from the cluster) or *gaps* (spaces).

The teacher reminded students that they could identify a representative or summary value for the entire distribution by choosing one value to represent their 6th-grade shoe size. Students readily suggested calculating the mean and using their calculators determined the mean shoe size to be 25 cm. Students were less inclined to recommend a *median* value as a summary/representative measure, and the teacher introduced the *median* as the exact middle value of the data set (the shoe size where half of the shoes are shorter in measure and half are longer).

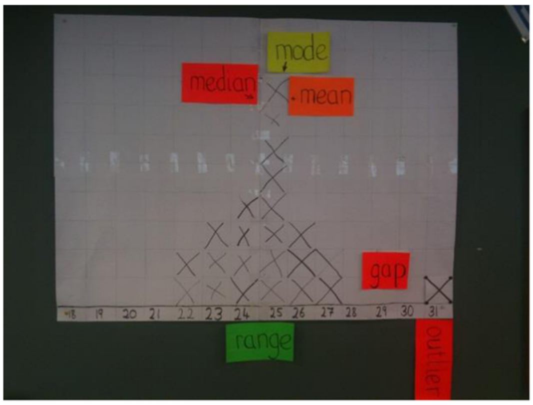

Fig. 5. Locating statistical measures on a distribution of data

Together with the teacher, the class calculated the median to be 25 cm.

The teacher highlighted that for their distribution of shoe sizes, the *mean*, *mode* and *median* are identical values (25 cm) because the distribution is

Box 5
Targeted statistical understanding: [CCSSM.6.SP.B.5D; UK: key stage 3]
If a student had a particular small/large shoe size, the influence of this value could be explored on each measure of central tendency leading to a discussion on the relationship between distributional shape and choice of summary measure.

almost symmetrical. She visually located these measures on the distribution (figure 5), noted their location in the centre of the distribution and discussed the suitability of 25 cm as an appropriate representative value. It should be noted that if the measures of central tendency were different in value, this would provide the opportunity for a discussion regarding the relationship between distributional shape and choice of measure of central tendency.

Children were supported in 'reading beyond the data' through questions such as '*Do you think a distribution for 2nd grade shoe lengths is similar?*'; '*Does this data accurately represent all possible 6th grades in this school?*'; '*Who might have a shoe length of 35 cm?*' Questions such as these provide the opportunity to engage in discussion about samples and populations and support students in making inferences from samples to populations. The teacher explored the consequences of an outlier (e.g. 35 cm) being included in the distribution and its impact on measures of central tendency, through questions such as '*Will the mode change? Will the mean change? Will the median change?*'.

PPDAC: conclusion

The teacher revealed that the actual shoe length found in the flowerbed was 31 cm in length. This value was then added to the distribution and explored in light of the class data, i.e. all students in this class can be eliminated as suspects because 31 cm lies apart from their data (figure 5). There is an opportunity here, once again, to explore the relationship between this sample and the population by discussing whether there may be a student in a different 6th-grade classroom who may have this shoe size.

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INVESTIGATION 2: RUGBY OR SOCCER? IDENTIFYING THE MYSTERY PLAYER

The focus of this investigation was to explore similarities and differences between the heights and weights of the national soccer and rugby teams. Comparing sports-player data requires a focus on distributional shape and supports students in understanding that measures of centre and variability (means, medians, modes and ranges) provide useful summaries of distributions, which can then be used to describe and compare data. The investigation can be easily modified to explore patterns and trends in data for State football or baseball teams.

PPDAC: problem

The teacher introduced the Mystery Player Problem. The goal of the investigation was to identify if a mystery player was a member of the national rugby or soccer squad. The only clues they would be provided were the weight and height of members of the national rugby and soccer squad. They would have to use this information to help predict to which team team the mystery player belongs.

PPDAC: plan

Students may collect data from sources such as the Internet or official sports publications and materials. We distributed player information cards (similar to baseball cards) which provided data about each member of the national rugby and soccer team (figure 6).

PPDAC: data

Students worked in groups to construct line plots of rugby player weight, rugby player height, soccer player weight and soccer player height (figure 6).

PPDAC: analysis

The teacher initially engaged students in 'reading the data' and 'reading between the data'. Each group prepared and made a brief presentation outlining three features of their distribution (figure 7). In general, groups reported on *minimum and maximum values* (e.g. shortest and tallest soccer players), the *range* of their data (e.g. difference between the lightest and heaviest rugby players), the most *frequently occurring* values (e.g. the most commonly occurring soccer player weight) and *interesting features* specific to their distribution (i.e. data landmarks such as particularly tall players).

A series of carefully constructed questions were asked to support students in the comparison of heights and weights for rugby and soccer players. Responses to questions requiring students to 'reading beyond the data' provided valuable insights into their statistical reasoning. When asked if there was a difference in height between soccer and rugby players, initial responses relied on reporting modes and on the variability of the data. Few students possessed functional understanding of means and medians, in other words they did not realize the value of using these measures of centre to compare the distributions.

Requiring students to identify differences between distributions highlighted the need to construct summary measures for each distribution. Students were encouraged to come up with one

Fig. 6. Plotting the height of the national rugby team

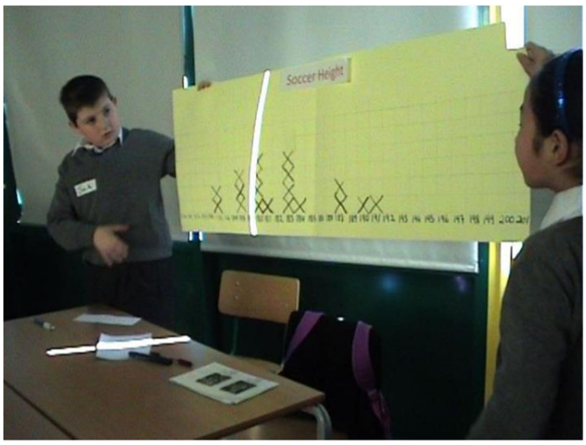

Fig. 7. Reporting on features of the distribution of soccer player heights

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Box 6: guided analysis of data

Reading the data

- What is the heaviest/lightest weight (minimum/maximum values) on the rugby team? On the soccer team?
- What is the tallest/shortest height (minimum/maximum values) in the soccer team? On the rugby team?
- What do you notice about the shape of the data? Are there clusters of data? Are there any outliers (unusual data value separated from the cluster)? Gaps (holes)?
- Are there any crossover points, i.e. common data values?

Reading between the data

- What is the difference between the heaviest and lightest player on the rugby team? On the soccer team?
- What is the difference between the tallest and shortest on the rugby? On the soccer team?
- What is the range in heights (or weight) of the rugby players?
- Is the range in heights (or weight) greater in the rugby team or soccer team?

Reading beyond the data
We encouraged students to examine the distributions of data to support their answers.

- Is there a difference in height between soccer and rugby players? How do you know?
- Is there a difference in weight between soccer and rugby players? How do you know?

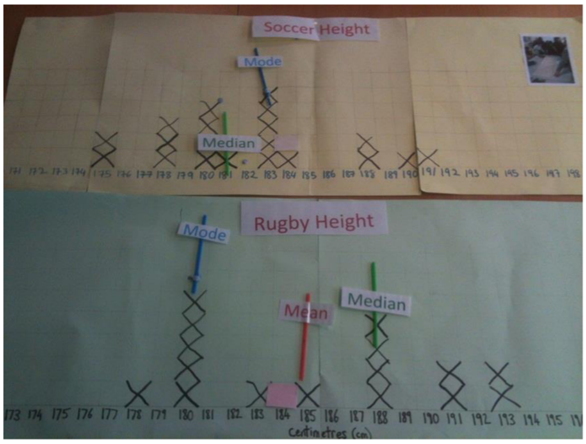

Fig. 8. Locating the mean, median and mode on a graph

value that best represented each distribution. While many identified modes and midranges as representative values, some students realized that the mean was an appropriate representative measure. We used this as an opportunity to discuss the functionality of the mean (and other measures of central tendency) as a summary measure for a distribution of data and to review understandings of these measures. Students then found the mean, median and mode of the distributions and located these measures (using signposts) onto the appropriate graphs (figure 8). Once the measures were located on the distributions, we engaged students in discussions of what the mean, median and mode each communicate about the distributions. For example, we asked: *Were any players (either rugby or soccer) the actual mean/median value? Why is the mean not in the middle of the graph? Why might the mean and median be different? Look at the median of the rugby height – what is the difference between this and the soccer height median? What does this difference tell us?*

We found that children are able to posit reasons to account for differences in the data. Some comments made were '*the mean, median and mode are higher for the weights of rugby players. This is because rugby players are generally bigger than soccer players*' and '*Rugby has more contact so they need more weight for more power*'. Students were required to support any hypotheses/assertions by making reference to the data. We encouraged this type of data-driven reasoning by continually asking students to justify their answers: '*Why do you say this? What data (on your graph) support your assertion?*'

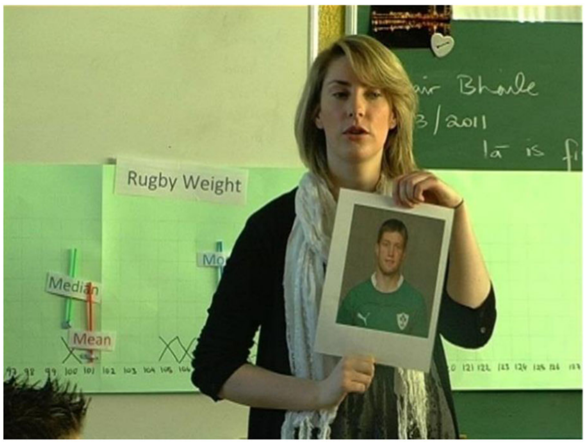

Fig. 9. Revealing the mystery player

PDDAC: conclusion

To conclude the PPDAC cycle, the teacher presented the body weight and height of the mystery player. She represented his body weight of 86 kg using a pink sticker on both soccer and rugby graphs (figure 8) and asked: *Based on what the graphs tells us about the general weight for a rugby/soccer player, could the mystery player belong to either of these teams? Which team is he most likely to belong to?* She then revealed that the mystery player was 184 cm tall and placed a sticker on both graphs to locate the height of the mystery player and asked: *Could he belong to either of the teams? Are his height/weight typical values for a rugby player? Soccer player? Which team do you think he belongs to? Why?* To conclude the lesson we revealed the face of the mystery player to be Ronan O'Gara (figure 9). This player belonged to the national rugby team.

CONCLUSION

The activities we describe can be easily modified to support the development of understandings of older students through a focus on larger data sets, a wider selection of graphical representations (such as stem-and-leaf plots or box-and-whisker plots) and a focus on additional statistical measures (for example, standard deviation). Furthermore, the use of technology can support older students in engaging in more exploratory analysis of larger data sets and thus increase the cognitive demands and expected learning outcomes.

Engaging students in these cycles of statistical investigation provide opportunities to see the functional use of graphs and statistical measures to display, summarize and compare distributions of data. Designing their own statistical investigations supports students in actively engaging in mathematics, in developing positive dispositions towards mathematics and facilitates the processes of mathematical inquiry. Our students were excited about the investigations and were actively trying to make sense of the data they themselves collected, thus fostering their own mathematical power, confidence and curiosity.

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