Comparing Means

One of the first statistical measures children encounter within the strand of Data is the (arithmetic) 'mean'. The mean is commonly called the average in primary classrooms.

The Primary School Mathematics Curriculum (PSMC 1990: 109) introduces the mean in 5th class. The procedure for finding the mean is relatively simple and is commonly referred to as 'add them all up and divide by the number of numbers'. Research indicates that the majority of children can compute the mean without understanding what the mean represents and when you might use it. The only reference to the mean in the PSMC is to the algorithmic procedure.

However, in order for pupils to make a smooth transition to secondary education particularly in light of the introduction of Project Maths, they need to really understand the concept of mean.

So, what is involved in understanding the mean? Three types of understanding is required, namely procedural understandings relating to the algorithm, conceptual understanding of what the mean represents, and functional understanding of why we use the mean (i.e. to summarise a set of data)

Research has shown that it is important to postpone the introduction of the algorithm until children have a conceptual understanding of the mean.

A classroom mathematical exploration of the mean

In this article, we outline a series of explorations with data, spanning between 3 and 5 lessons, to develop conceptual understanding of the mean. These explorations were field tested with 5th class children in Limerick city schools.

While various real contexts could be used, this article focuses on an activity which examines the accuracy of the assertion that a person's foot is the same length as their forearm.

Lesson 1

Prior to starting these explorations, we have found it useful to (a) revise the procedure for rounding decimal numbers, and (b) place children in even groups (e.g. 4 or 6 in a group). From our experience, we found that steps 1-8 take one class period, steps 9-10 a second, and 11-14 a third class period.

Present the claim: The length of the human foot is the same length as the human forearm.

Discuss the claim. Is the claim true or false? Why? Have children record their beliefs.

Physical determination of the mean to test the assertion

The first lesson focuses on determining the mean value for the length of a foot (in each group) through the following sequence of activities:

1. Explain that each group will explore whether, for their group, the length of their feet is the same as the length of their forearms. Present each child with two strips of paper.
2. Have each child measure their foot and cut a strip of paper that matches the length of their foot.
3. In groups, record and discuss the different lengths. What was the shortest/longest foot?
4. Encourage each group to come up with one value that fairly represents the

Image 1: Taping the individual paper strips representing the foot length of each group

Image 2: Comparing the paper strips representing the mean foot and forearm length for a group

length of a foot in their group. We call this a representative value. Have each group record their representative value and be prepared to share and justify their particular value. [We found children tend to use the most frequently occurring value (i.e. the mode) or a value that falls in the middle of the range of values (i.e. the midrange)].

5. Explain that they are going to find a representative value for their group by levelling out the values for their individual foot lengths.

6. In each group have children tape their foot strips end to end (see image 1). Take the long taped strip and fold it into the number of pieces equal to the number of children in the group (i.e. into four pieces of equal length if there are four in the group).

7. The length of each equal piece is the mean.

8. Extension/discussion: Children can compare their individual foot length to the mean value and discuss why there are some foot lengths above and below the mean.

Lesson 2

The next stage, usually the second lesson, focuses on determining the mean length of the forearm in order to facilitate children in testing the claim that the foot is the same length as the forearm.

9. Repeat steps 1-8 this time focusing on the length of the forearm.

10. Each group now has a mean group value for their feet and their forearms. Using the strips compare the values (see image 2) and ascertain if the original claim was true: In your group, is the mean foot length equal to the mean arm length? Was their original prediction correct?

[You may find that some groups find the means to be similar whereas others will find different means. Discuss why differences between groups occur (measurement and rounding errors etc.)] See Image 2.

Lesson 3: Introducing the mean algorithm

The third lesson focuses on helping children extend from their group sample to the whole class data.

11. Discuss whether the claim might be true for the entire class. Is the mean foot length equal to the mean arm length for the entire class?

12. Ask how we might find the mean foot/forearm length for the entire class. Would the paper strip method be accurate? Easy to manage? Why? Why not?

At this stage of the discussion in each of

the classrooms we visited, there was at least one child who suggested that, rather than use the paper strip method, it would be easier to add up the [foot/forearm] lengths for everyone in the class and divide by the number of people in the class. If your class are not reaching this conclusion by themselves (allow a little time!), try the next step to support the children in linking the mean algorithm to the activities carried out.

13. Tell the children that that you are going to imagine finding the mean for the entire class using the paper strip method. Imagine making the long paper strip for everyone in the class (for example 28 children). How long would the strip be? How would you find out? The total length would be the sum of the lengths of each individuals foot/forearm. Imagine dividing the long strip into 28 equal pieces. To find the length of one section after the strip was divided into 28 equal pieces, divide the total length by 28.

14. Using calculators, find the mean foot and forearm length for the entire class. Is the claim true?

Why use this method to explore the mean?

Too often we launch into teaching the mean by introducing the algorithm and never discuss why we might want to find

the mean of a data set. This exploration helps children see the mean as one single value that can be used to represent a set of numbers. It also identifies the mean as a useful measure to use if we want to compare data sets. Determining the mean using the paper strip method appeals to all types of learners as it gives a hands-on and visual way to experience the mean of a data set . This method is a natural precursor to the algorithm. This approach also provides a context that children can draw upon when exploring the properties of the mean (in particular properties 1-4 outlined on the web version of this article).

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Web version: An extended version of this article including transcripts of classroom discussion is available online.

Is your foot really the same length as your forearm?